Throughout your study of science, you will often need to solve math problems. This appendix is designed to help you quickly review the basic math skills you will use most often.

Fractions

Adding and Subtracting Fractions

To add or subtract fractions that have the same denominator, add or subtract the numerators, and then write the sum or difference over the denominator. Express the answer in lowest terms.

Examples

$$\frac{3}{10} + \frac{1}{10} = \frac{3+1}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$$

To add or subtract fractions with different denominators, find the least common denominator. Write an equivalent fraction for each fraction using the least common denominator. Then add or subtract the numerators. Write the sum or difference over the least common denominator and express the answer in lowest terms.

Examples

$$\frac{1}{3} + \frac{3}{5} = \frac{5}{15} + \frac{9}{15} = \frac{5+9}{15} = \frac{14}{15}$$

$$\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{7-2}{8} = \frac{5}{8}$$

Multiplying Fractions

When multiplying two fractions, multiply the numerators to find the product's numerator. Then multiply the denominators to find the product's denominator. It helps to divide any numerator or denominator by the greatest common factor before multiplying. Express the answer in lowest terms.

Examples

$$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

$$\frac{4}{14} \times \frac{6}{9} = \frac{2 \times 2}{7 \times 2} \times \frac{2 \times 3}{3 \times 3} = \frac{2 \times 2}{7 \times 3} = \frac{4}{21}$$

Dividing Fractions

To divide one fraction by another, invert and multiply. Express the answer in lowest terms.

Examples

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

$$\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{9 \times 1}{2 \times 5} = \frac{9}{10}$$

Ratios and Proportions

A ratio compares two numbers or quantities. A ratio is often written as a fraction expressed in lowest terms. A ratio also may be written with a colon.

Examples

The ratio of 3 to 4 is written as 3 to 4, $\frac{3}{4}$, or 3 : 4.

The ratio of 10 to 5 is written as $\frac{10}{5} = \frac{2}{1}$, or 2 : 1.

A proportion is a mathematical sentence that states that two ratios are equivalent. To write a proportion, place an equal sign between the two equivalent ratios.

Examples

The ratio of 6 to 9 is the same as the ratio of 8 to 12.

$$\frac{6}{9} = \frac{8}{12}$$

The ratio of 2 to 4 is the same as the ratio of 7 to 14.

$$\frac{2}{4} = \frac{7}{14}$$

You can set up a proportion to determine an unknown quantity. Use x to represent the unknown. To find the value of x, cross multiply and then divide both sides of the equation by the number that comes before x.

Example

Two out of five students have blue notebooks. If this same ratio exists in a class of twenty students, how many students in the class have blue notebooks?

$$\frac{2}{5} = \frac{x}{20} \qquad \leftarrow \textbf{Cross multiply.}$$
$$2 \times 20 = 5x \qquad \leftarrow \textbf{Divide.}$$
$$8 = x$$

Percents and Decimals

To convert a percent to a decimal value, write the number without the percent sign and move the decimal point two places to the left. Add a zero before the decimal point.

Examples

$$38\% = 0.38$$
$$13.92\% = 0.1392$$

You can convert a decimal value to a percent value by moving the decimal point two places to the right and adding the percent sign.

Examples

$$0.46 = 46\%$$
$$0.8215 = 82.15\%$$

Exponents

A base is a number that is used as a factor. An exponent is a number that tells how many times the base is to be used as a factor.

Example

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

A power is any number that can be expressed as a product in which all of the factors are the same. Any number raised to the zero power is 1. Any number raised to the first power is that number. The only exception is the number 0, which is zero regardless of the power it is raised to.

Exponents	
Powers of 2	**Powers of 10**
$2^2 = 4$	$10^2 = 100$
$2^1 = 2$	$10^1 = 10$
$2^0 = 1$	$10^0 = 1$
$2^{-1} = \frac{1}{2}$	$10^{-1} = \frac{1}{10}$
$2^{-2} = \frac{1}{4}$	$10^{-2} = \frac{1}{100}$

Multiplying Exponents

To multiply exponential expressions with the same base, add the exponents. The general expression for exponents with the same base is $x^a \times x^b = x^{a+b}$.

Example

$$3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^6 = 729$$

To raise a power to a power, keep the base and multiply the exponents. The general expression is $(x^a)^b = x^{ab}$.

Example

$$(3^2)^3 = (3^2) \times (3^2) \times (3^2) = 3^6 = 729$$

To raise a product to a power, raise each factor to the power. The general expression is $(xy)^n = x^n y^n$.

Example

$$(3 \times 9)^2 = 3^2 \times 9^2 = 9 \times 81 = 729$$

Dividing Exponents

To divide exponential expressions with the same base, keep the base and subtract the exponents. The general expression is:

$$\frac{x^a}{x^b} = x^{a-b}$$

Example

$$\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$$

When the exponent of the denominator is greater than the exponent of the numerator, the exponent of the result is negative. A negative exponent follows the general expression:

$$x^{-n} = \frac{1}{x^n}$$

Example

$$2^3 \div 2^5 = 2^{3-5} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Scientific Notation

Scientific notation is used to express very large numbers or very small numbers. To convert a large number to scientific notation, move the decimal point to the left until it is located to the right of the first nonzero number. The number of places that you move the decimal point becomes the positive exponent of 10.

Example

$18{,}930{,}000 = 1.893 \times 10^7$

To write a number less than 1 in scientific notation, move the decimal point to the right of the first nonzero number. Use the number of places you moved the decimal point as the negative exponent of 10.

Example

$0.0027 = \dfrac{2.7}{10 \times 10 \times 10} = 2.7 \times 10^{-3}$

Adding and Subtracting

To add or subtract numbers in scientific notation, the exponents must be the same. If they are different, rewrite one of the numbers to make the exponents the same. Then write the answer so that only one number is to the left of the decimal point.

Example

$3.20 \times 10^3 + 5.1 \times 10^2$

$$= 32.0 \times 10^2 + 5.1 \times 10^2$$
$$= 37.1 \times 10^2$$
$$= 3.71 \times 10^3$$

Multiplying and Dividing

To multiply or divide numbers in scientific notation, the exponents are added or subtracted.

Examples

$$(1.2 \times 10^3) \times (3.4 \times 10^4) = (4.1 \times 10^{3 + 4})$$
$$= 4.1 \times 10^7$$
$$(5.0 \times 10^9) \div (2.5 \times 10^6) = (2.0 \times 10^{9 - 6})$$
$$= 2.0 \times 10^3$$

Significant Figures

When measurements are combined in calculations, the uncertainty of each measurement must be correctly reflected in the final result. The digits that are accurate in the answer are called significant figures. When the result of a calculation has more significant figures than needed, the result must be rounded off. If the first digit after the last significant digit is less than 5, round down. If the first digit after the last significant digit is 5 or more, round up.

Examples

1577 rounded to three significant figures is 1580.
1574 rounded to three significant figures is 1570.
2.458462 rounded to three significant figures is 2.46.
2.458462 rounded to four significant figures is 2.458.

Adding and Subtracting

In addition and subtraction, the number of significant figures in the answer depends on the number with the largest uncertainty.

Example

$$
\begin{array}{r}
25.34 \text{ g} \\
152 \text{ g} \\
+ \quad 4.009 \text{ g} \\
\hline
181 \text{ g}
\end{array}
$$

The measurement with the largest uncertainty is 152 g and it is measured to the nearest gram. Therefore, the answer is given to the nearest gram.

Multiplying and Dividing

In multiplication and division, the measurement with the smallest number of significant figures determines the number of significant figures in the answer.

Example

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$
$$= \frac{20.79 \text{ g}}{5.5 \text{ mL}}$$
$$= 3.8 \text{ g/mL}$$

Because 5.5 mL has only two significant figures, the answer must be rounded to two significant figures.

Formulas and Equations

An equation is a mathematical sentence that contains one or more variables and one or more mathematical operators (such as $+$, $-$, $\div$, $\times$, and $=$). An equation expresses a relationship between two or more quantities.

A formula is a special kind of equation. A formula such as $V = l \times w \times h$ states the relationship between unknown quantities represented by the variables V, l, w, and h. The formula means that volume (of a rectangular solid) equals length times width times height. Some formulas have numbers that do not vary, such as the formula for the perimeter of a square: $P = 4s$. In this formula, the number 4 is a constant.

To solve for a quantity in an equation or formula, substitute known values for the variables. Be sure to include units.

Example

An airplane travels in a straight line at a speed of 600 km/h. How far does it fly in 3.5 hours?

Write the formula that relates speed, distance, and time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$
$$v = \frac{d}{t}$$

To solve for distance, multiply both sides of the equation by t.

$$v = \frac{d}{t}$$
$$v \times t = \frac{d}{t} \times t$$
$$v \times t = d$$

Substitute in the known values.

$$600 \text{ km/h} \times 3.5 \text{ h} = d$$
$$d = 2100 \text{ km}$$

Conversion Factors

Many problems involve converting measurements from one unit to another. You can convert units by using an equation that shows how units are related. For example, 1 in. $= 2.54$ cm relates inches and centimeters.

To write a conversion factor, divide both sides of the equation by 1 in.

$$\frac{1 \text{ in.}}{1 \text{ in.}} = \frac{2.54 \text{ cm}}{1 \text{ in.}}$$
$$1 = 2.54 \text{ cm/in.}$$

Because the conversion factor is equal to 1, you can multiply one side of an equation by it and preserve equality. You can make a second conversion factor by dividing both sides of the equation by 2.54 cm.

$$\frac{1 \text{ in.}}{2.54 \text{ cm}} = \frac{2.54 \text{ cm}}{2.54 \text{ cm}} = 1$$

One conversion factor converts inches to centimeters and the other converts centimeters to inches. Choose the conversion factor that cancels out the unit that you have a measurement for.

Example

Convert 25 inches to centimeters. Use d to represent the unknown number of centimeters.

$$d = 25 \text{ in.} \times \frac{2.54 \text{ cm}}{1 \text{ in.}}$$
$$= 64 \text{ cm}$$

Some conversions are more complicated and require multiple steps.

Example

Convert 23°F to a Celsius temperature.

The conversion formula is
$$°F = (\frac{9}{5} \times °C) + 32°F$$

Substitute in 23°F:

$$23°F = (\frac{9}{5} \times °C) + 32°F$$
$$23°F - 32°F = \frac{9}{5} \times °C$$
$$-9°F = \frac{9}{5} \times °C$$
$$-9°F \times \frac{5}{9} = -5°C$$
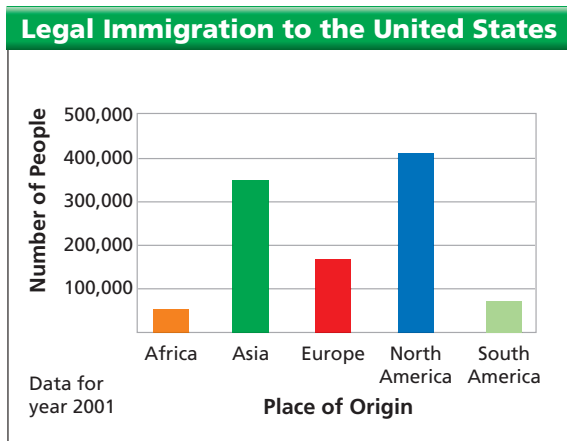
Math Skills

Data Tables

Data tables help to organize data and make it easier to see patterns in data. If you plan data tables before doing an experiment, they will help you record observations in an orderly fashion.

The data table below shows United States immigration data for the year 2001. Always include units of measurement so people can understand the data.

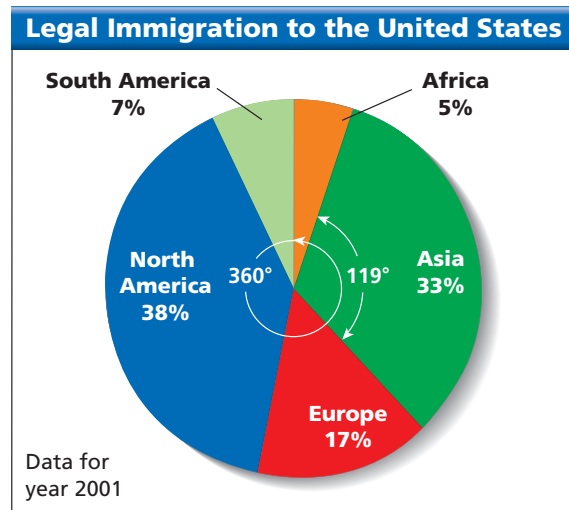| Immigration to the United States, 2001	
Place of Origin	**Number of Legal Immigrants**
Africa	53,948
Asia	349,776
Europe	175,371
North America	407,888
South America	68,888

Bar Graphs

To make a bar graph, begin by placing category labels along the bottom axis. Add an overall label for the axis *Place of Origin.* Decide on a scale for the vertical axis. An appropriate scale for the data in the table is 0 to 500,000. Label the vertical axis *Number of People.* For each continent, draw a bar whose height corresponds to the number of immigrants. You will need to round off the values. For example, the bar for Africa should correspond to 54,000 people. Add a graph title to make it clear what the graph shows.

Legal Immigration to the United States

Data for year 2001

Circle Graphs

Use the total number to calculate percentages. For example, the percentage of immigrants from Africa in 2001 was 53,948 ÷ 1,061,984 = 0.051 ≈ 5%. Multiply each percent by 360° to find the central angle of each wedge. For Africa, the central angle is 18°. Use a protractor to draw each central angle. Color and label the wedges and finish your graph with a title.

Legal Immigration to the United States

Data for year 2001

Line Graphs

The slope of a straight-line graph equals the "rise over the run." The rise is the change in the *y* values and the run is the change in the *x* values. Using points A and B on the graph below gives

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{5-3}{9-3} = \frac{2}{6} = 0.33$$

Slope of a Straight-Line Graph